Name of College. - S.S. College. J. Bad

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Date 07-07-2020

Deptt - Mathematics

Topic - Tangents and Normals (Differential Calculus)

Class - B.Sc I (HONS)

Period - I + II.

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1. Tangents and Normals are st. line and perpendicular to each other.

i.e slope of Tangent × slope of Normal = -1

$$\text{Slope of Normal} = -\frac{1}{\text{slope of tangent}}$$

2. If $y = f(x)$ be the equation of curve then $\left(\frac{dy}{dx}\right)_{(x,y)}$ represent the slope of tangent at (x,y) where Tangent is drawn at (x,y).

3. Equation of tangent at (x_1,y_1) when the equation of curve is $y = f(x)$

$$\frac{y-y_1}{x-x_1} = \left(\frac{dy}{dx}\right)_{(x_1,y_1)}$$

4. Equation of Normal at (x_1,y_1) is given by

$$\frac{y-y_1}{x-x_1} = -\frac{1}{\left(dy/dx\right)_{(x_1,y_1)}}$$

5. If Equation of curve is $f(x,y) = 0$

then Equation of tangent at (x_1, y_1) is

$$(X - x_1) f_x + (Y - y_1) f_y = 0$$

Also Equation of Normal

$$\frac{Y - y_1}{X - x_1} = \frac{1}{\left(\frac{dy}{dx}\right)_{x_1, y_1}}$$

$$\Rightarrow \frac{X - x_1}{f_x} = \frac{Y - y_1}{f_y}$$

6. Slope of the tangent when Equation of curve is $f(x,y) = 0$

$$= -\frac{f_x}{f_y}$$

and slope of Norm $= \frac{f_y}{f_x}$

* Show that

(i) The Cartesian subtangent $= \frac{y}{y_1}$

(ii) The Cartesian subnormal $= y y_1$

(iii) The length of the tangent $= \frac{y}{y_1}\sqrt{1+y_1^2}$

(iv) The length of the Normal $= y\sqrt{1+y_1^2}$

Where $y_1 = \frac{dy}{dx}$

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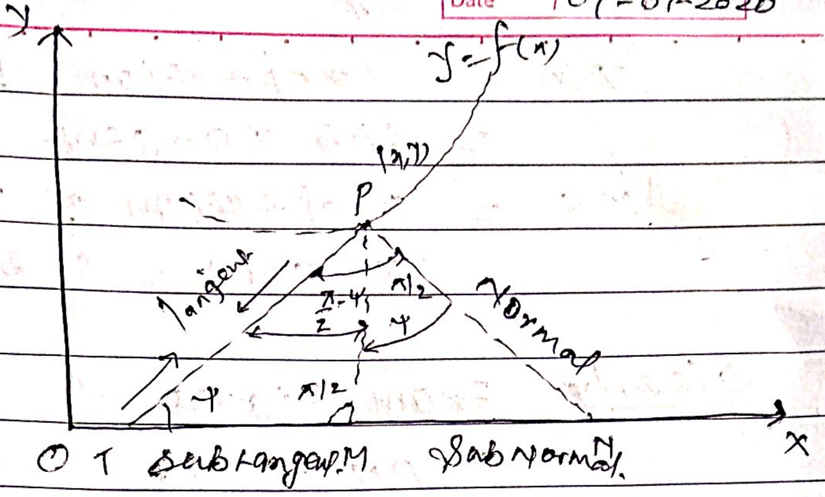
Idea's on :→



Tangent → the length between point of contact and the point where tangent intersect x-axis

Subtangent — Projection of tangent on x-axis

Normal → the length of normal is between the foot of Normal and the point of intersection of normal where it intersect x-axis

Sub-Normal → Projection of Normal on x-axis.

for Above

Now let $y = f(x)$ be the equation of curve. Let $P(x, y)$ be any point on it.

Draw PT Tangent at P which intersect x-axis at T and PN is the normal drawn at P.

Here ~~let~~ $\angle PTN = \psi$ = slope of the Tangent

Clearly $\tan\psi = \dfrac{dy}{dx}$

Here $PM = y$

TM = projection of PT on x-axis
= Sub Tangent.

MN = projection of normal PN on x-axis = Sub Normal

Clearly from figure

$\angle TPN = \pi/2$ $\quad$ $\angle PMN = \pi/2$

$\angle PMT = \pi/2$ $\quad$ $\angle MPN = \frac{\pi}{2} - \left(\frac{\pi}{2} - \psi\right) = \psi$

$\angle PTM = \psi$

$\angle TPM = \pi/2 - \psi$

Now from right-angled $\Delta\, PTM$

~~[illegible]~~ $\frac{MP}{PT} = \sin\psi$

$\Rightarrow PT = \frac{MP}{\sin\psi} = y \operatorname{cosec}\psi$

$= y\sqrt{1+\cot^2\psi}$

Since $\tan\psi = \frac{dy}{dx} = y_1$

$= y\sqrt{1+\frac{1}{\tan^2\psi}}$

$= y\sqrt{\frac{1+\tan^2\psi}{\tan\psi}}$

$= \frac{y\sqrt{1+y_1^2}}{y_1}$

$\therefore$ Length of Tangent $= \frac{y\sqrt{1+y_1^2}}{y_1}$

for length of Normal

From right angled triangle PMN

$\angle MPN = \psi$

$\therefore \frac{MP}{PN} = \cos\psi$

$\Rightarrow PN = \frac{MP}{\cos\psi} = MP\sec\psi$

$= y\sec\psi$

$= y\sqrt{1+\tan^2\psi}$

$= y\sqrt{1+y_1^2}$

for Sub Tangent

From right angled Δ PMT

$\angle PTM = \psi$

$\frac{TM}{PM} = \cot\psi$

$\Rightarrow TM = PM\cot\psi$

$= y \cdot \frac{1}{\tan\psi} = \frac{y}{y_1}$

for Sub Normal

From right angled Δ PMN

$\angle MPN = \psi$

Now $\frac{PM}{MN} = \cot\psi$

$\Rightarrow MN = PM\tan\psi$

$= y y_1$

Thus Length of Tangent $= \frac{y\sqrt{1+y_1^2}}{y_1}$

Length of Normal $= y\sqrt{1+y_1^2}$

Sub tangent $= \frac{y}{y_1}$

Subnormal $= y y_1$. Where $y_1 = \left(\frac{dy}{dx}\right)_{x}$.